Math 434 Assignment 2

Due March 15

Assignments will be collected in class.

2. For κ, λ infinite cardinals with $\lambda < \kappa$ prove that $|\{x \subseteq \kappa : |x| = \lambda\}| = \kappa^\lambda$.

   Solution: κ^λ is the cardinality of the set of functions $\lambda \to \kappa$.

   Each $x \subseteq \kappa$ with $|x| = \lambda$ induces a map $\lambda \to \kappa$ with range x, so $|\{x \subseteq \kappa : |x| = \lambda\}| \leq \kappa^\lambda$.

   Now note that $\kappa = \lambda \times \kappa$, and each function $f\colon \lambda \to \kappa$ is a subset of $\lambda \times \kappa$ of size λ. So $|\{x \subseteq \kappa : |x| = \lambda\}| = |\{x \subseteq \lambda \times \kappa : |x| = \lambda\}| \geq \kappa^\lambda$.

4. Prove that $2^{\aleph_0} \neq \aleph_\omega$.

   Solution: Suppose that $2^{\aleph_0} = \aleph_\omega$. Then $cof(2^{\aleph_0}) = \aleph_0$. So

   $$2^{\aleph_0} < (2^{\aleph_0})^{cof(2^{\aleph_0})} = (2^{\aleph_0})^{\aleph_0} = 2^{\aleph_0 \cdot \aleph_0} = 2^{\aleph_0}.$$

   This is a contradiction.